Phys 410
Fall 2015
Lecture #1 Summary
1 September, 2015

We will cover three formulations of classical mechanics in this class: Newtonian, Lagrangian and Hamiltonian, as well as extensions of mechanics to include relativistic and chaotic aspects of particle motion. After a general introduction and overview of classical mechanics, we talked about basic properties of space and time, including the use of vectors and their derivatives, typically taken with respect to time. We also reviewed the scalar and vector products of two vectors. The Cartesian coordinate system is particularly convenient because the 3 unit vectors $(\hat{\imath}, \hat{\jmath}, \hat{k})$ do not vary in direction as the position vector $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$ evolves in time *t*. This is not the case for other coordinate systems such as polar, cylindrical, spherical, etc.

We shall assume that time *t* evolves smoothly and that all observers agree on timing and the evolution of time. This assumption will be re-examined later when we discuss relativity. Also, the uniformity of time means that we can freely chose the zero of time to be anywhere in the evolution of a system.

A reference frame is a choice of origin, a coordinate system, coordinate axes and directions, and a choice for the origin of time. The physics that we describe should be independent of the choice of reference frame, as long as it is not accelerating, or compared to another relevant reference frame moving at speeds approaching that of light. These two qualifications will be examined in more detail later.

Mass is a measure of an object's inertia, or resistance to acceleration. It is also the 'gravitational charge' of an object. This coincidence is known as the Principle of Equivalence. Mass is measured in kilograms (kg).

A force is an influence that produces acceleration of an object. Its direction is the direction of the resulting acceleration. A force of 1 Newton will produce an acceleration of 1 m/s^2 of a 1 kg mass.

We discussed Newton's Laws of motion for point-like particles. The first law states that an object that is subjected to zero net force will move with constant velocity. The second law says that the net force acting on the object will always equal the mass times the acceleration of the object. The third law says that forces always occur in pairs.

For a point-like particle that does not change its mass while in motion, the second law can be stated in terms of the linear momentum of the particle, $\vec{p} = m\vec{v}$ as $\vec{F}_{net} = \dot{\vec{p}}$, where the "dot" denotes derivative with respect to time.

The first law is actually contained as a special case of the second law. So why call it out explicitly? The answer is that the first law helps us to distinguish frames of reference that are 'inertial' from those that are 'non-inertial.' An inertial reference frame is one in which Newton's first law holds true. If we prepare a particle such that it has zero net force and make the kinematic observation that it moves with constant velocity, then we know that we are in an inertial reference frame. As a counter-example, we considered the rotating (accelerating) frame of reference on the surface of a rotating table (while on the rotating table you experience centripetal acceleration). For example, consider the reference frame attached to the table with the z-axis coincident with the axis of rotation of the table. An observer in this frame would witness an object subjected to zero net force undergo acceleration, i.e. a change in the direction and/or magnitude of its velocity vector as a function of time. This was illustrated with a friction-less ice cube moving on the surface. By the way, we will later 'patch up' Newton's laws of motion to work in non-inertial reference frames by adding new forces to account for the accelerated nature of the reference frame. This turns out to be very convenient for doing physics because we often find ourselves working in non-inertial reference frames.

A consequence of Newton's third law of motion is the possibility of conserving the total momentum of a collection of particles interacting with each other with arbitrary forces. If the net external force on a set of N interacting particles is zero, the total momentum of that system, $\vec{P} = \sum_{\propto=1}^{N} \vec{p}_{\propto}$, is conserved. As an example, consider two particles colliding, but subject to no net external force. If they collide elastically, then both energy and momentum of the two-particle system are conserved. If they collide perfectly inelastically (i.e. they collide and stick together) then mechanical energy is not conserved, but the total momentum IS conserved. This robust conservation law is very useful in many branches of physics.

We reviewed the 6-step process for solving problems involving Newton's laws of motion, and the process for solving problems involving conservation laws. The statement that $\vec{a} = \vec{F}_{net}/m$, where $\vec{a}$ is the acceleration, and $\vec{F}_{net}$ is the net force acting on the object of mass m, is a bridge between the world of kinematics (the description of motion) on the left and the world of dynamics (understanding *why* motion occurs) on the right. It is also a statement of cause ($\vec{F}_{net}$) and effect ($\vec{a}$).